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# AN ANALYSIS OF MACHINE TOOL SPEED-BOXES WITH MINIMUM NUMBER OF GEARS

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*In this paper, various aspects of machine tool speed-boxes with minimum number of gears for 6, 8 and 9 speeds have been analysed. A survey has shown that very rarely these gear-boxes have been used. The analysis has shown that 6 speed-gear boxes of this type have definite superiority and others have limited applications. Other aspects like sizes of shafts and gears have been discussed. Complete procedure for analysis for one case has been presented. Final results for the other cases have been included. A designer intending to use these gear-boxes can use these results directly.*

## NOTATIONS

A, B, C, D = shafts in succession from input, to output end

$a_1, a_2, a_3$ = sizes of gears (either number of teeth or pitch diameters) on shaft A in the increasing order

$b_1, b_2, b_3$; $c_1, c_2, c_3$; $d_1, d_2, d_3$ = sizes of gears on shafts B, C and D respectively. Gears with corresponding subscripts mesh with each other

$S$ = the ratio of the smallest output speed to the input speed

$S_{max}$ = maximum permissible value of $S$, beyond which the gear sizes become negative

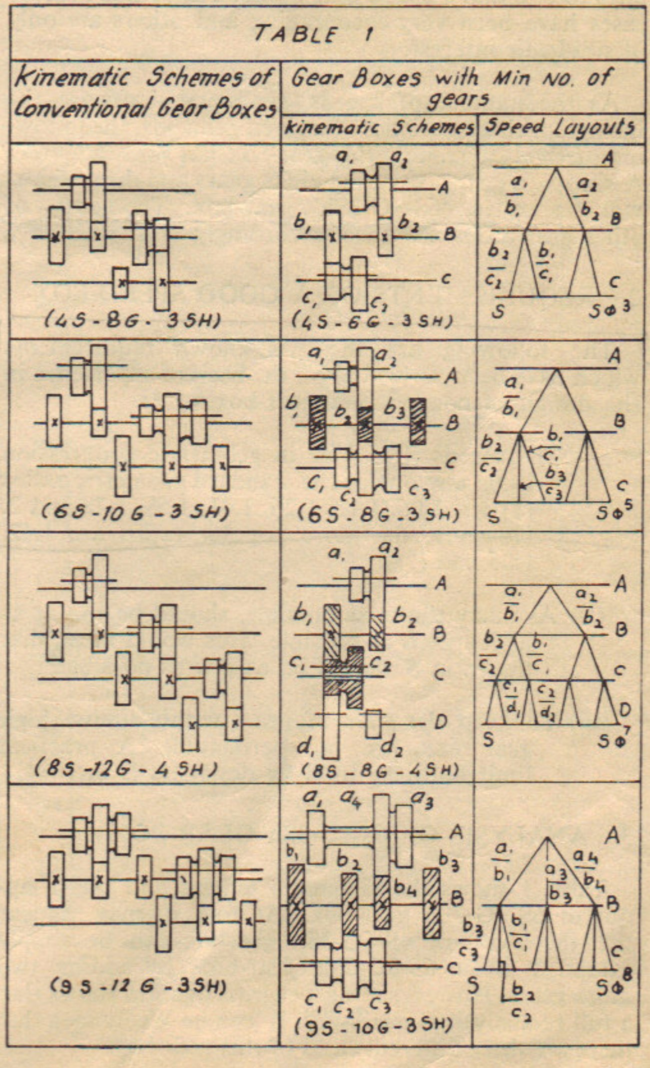
$S_{opt}$ = value of $S$ at which $i_{max}$ in a given gear-box becomes smallest

$i$ = the ratio of the size of a given gear to that of the smallest in the gear-box

$i_{max}$ = the ratio of the size of the biggest gear to that of the smallest in the optimum design of gear-box

## 1. INTRODUCTION

The endeavour of any machine tool speed-box designer is to use as few a number of gears as possible. The number of gears affect the size of a gear-box in many ways *viz.* the space occupied by the gears themselves, lengths of shafts, static weights and forces of unbalance on the shafts and bearings, diameters of the shafts, the number and complexity of locking and gear changing mechanisms, etc. A reduction of even one gear can result in a significant decrease in the size and complexity of the gear-box. However, as the number of gears is sought to be reduced, the analysis of the gear-box becomes more and more complicated. Further it is known that there



is a theoretical lower limit to the number of gears in a gear-box with a definite number of output speeds in geometric progression.

Table 1 shows the kinematic schemes of conventional gear-boxes for 4, 6, 8 and 9 speeds and the kinematic schemes and speed layouts for corresponding gear boxes with minimum number of gears. The main difference between the two types is that in the former, any gear can mesh with only one gear, whereas in the latter type, some gears (shown hatched) can mesh simultaneously with two gears on two shafts. Such gears are usually called connected wheels. Table 1 shows open gear-boxes with the maximum possible number of connected wheels in each case resulting in a minimum number of gears. The word 'minimum' is used here in this sense. It may be noted that the schemes in Table 1 are simplified for convenience of analysis. The gears on the first shaft are arranged in the order of their sizes *i.e.* $a_1 < a_2 < a_3$.

A survey conducted by the authors of about hundred kinematic schemes of Russian machine tools has revealed that the use of gear-boxes with minimum number of gears is extremely rare. Further, none of the standard hand books on Machine Tool Design has referred to the analysis of these gear-boxes. G. White and D. J. Sanger[1] have made a thorough analysis of the four-speed gear-box with six gears, very recently. The techniques used by them in the analysis are very interesting. In the present paper, the authors have extended these techniques to 6, 8 and 9 speed gear-boxes. The results in some cases have been very encouraging and others are only of academic interest.

As the number of speeds, gears and shafts are the important characteristics of a given gear-box, the following method of naming the gear-box has been adopted. A six-speed-gear-box, using eight gears and three shafts will be called '6S-8G-3Sh' gear-box. The names of other gear-boxes are given accordingly in Table 1.

## 2. REQUIREMENTS OF A GOOD SPEED-BOX

The following are the well-known requirements which have been arrived at on the basis of experience in the design of conventional gear-boxes [2-4]:

(i) The speeds should be in geometric progression using any one of the standard geometric ratios ($\phi$) *viz.* 1.06, 1.12, 1.26, 1.41, 1.58, 1.78 and 2. However, the more popular ratios are 1.12, 1.26 and 1.41.

(ii) As many shafts as possible, should be run at as high speeds as possible. This would mean that the ratio $S$ should be as low as possible.

(iii) To keep the sizes of gears within limits, high gear sizes are not permissible. A practical limit normally used by designers is $i_{max}=4$.

## 3. ANALYSIS OF 6S-8G-3Sh GEAR-BOX

Table 1 shows the kinematic scheme and speed layout of 6S-8G-3Sh gear-box. At first it may appear that the results of 4S-6G-3Sh gearbox can be simply extended to a 6S-8G-3Sh gear-box, by adding two more gears. However, a deeper investigation shows that a full re-analysis is necessary. It can be easily seen that there are three different cases of this gear-box.

Case 1: $\frac{b_3}{c_3}$ is the smallest gear ratio between shafts B and C *i.e.*, $\frac{b_1}{c_1} > \frac{b_2}{c_2} > \frac{b_3}{c_3}$.

Case 2: $\frac{b_3}{c_3}$ is the biggest gear ratio between shafts B and C *i.e.* $\frac{b_3}{c_3} > \frac{b_1}{c_1} > \frac{b_2}{c_2}$

Case 3: $\frac{b_3}{c_3}$ is the intermediate gear ratio between shafts B and C *i.e.*, $\frac{b_1}{c_1} > \frac{b_3}{c_3} > \frac{b_2}{c_2}$

The kinematic scheme and speed layout for the Case 3 are shown in Table 1, whereas the same for Cases 1 and 2 are shown in Fig. 1. Actually there is another

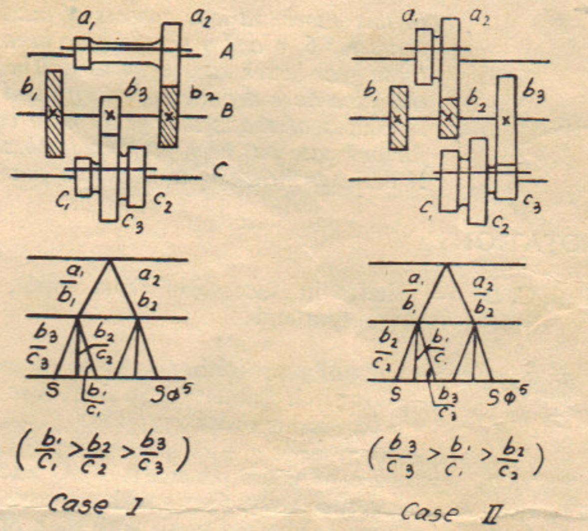



**Fig. 1**
**Kinematic schemes and speed layouts of 6S - 8G - 3SH gear boxes**

set of cases where three gear pairs exist between shafts A and B and two gear pairs between shafts B and C. This type is not discussed here as it is customary in gear-box design to put higher number of gear pairs between the final shafts. On analysis, it has been observed that only Case 3 is of practical use and therefore its analysis is presented in detail below. From the kinematic scheme and speed layout diagram of 6S-8G-3Sh Case 3 gear-box, as shown in Table 1, the following equations can be written down:

$$S = \frac{a_1}{b_1} \cdot \frac{b_2}{c_2} \tag{1}$$

$$S\phi = \frac{a_1}{b_1} \cdot \frac{b_3}{c_3} \tag{2}$$

$$S\phi^2 = \frac{a_1}{b_1} \cdot \frac{b_1}{c_1} = \frac{a_1}{c_1} \tag{3}$$

$$S\phi^3 = \frac{a_2}{b_2} \cdot \frac{b_2}{c_2} = \frac{a_2}{c_2} \tag{4}$$

$$S\phi^4 = \frac{a_2}{b_2} \cdot \frac{b_3}{c_3} \tag{5}$$

$$S\phi^5 = \frac{a_2}{b_2} \cdot \frac{b_1}{c_1} \tag{6}$$

From the condition of constancy of the centre distances between two shafts, we get

$$a_1+b_1=a_2+b_2 \tag{7}$$

$$b_1+c_1=b_2+c_2 \tag{8}$$

$$b_2+c_2=b_3+c_3 \tag{9}$$

It will be desirable to determine the sizes of gears $a_1, b_1$........etc.in terms of $S$ and $\phi$. For the sake of generality, the sizes of all the gears are referred to in terms of the smallest gear $a_1$ on the shaft A *i.e.*, $\frac{a_2}{a_1}, \frac{b_1}{a_1}$ ...etc. Thus there are three independent parameters $S$, $\phi$ and $a_1$. It is seen from the above that there are 9 available equations for 7 unknowns. So the gearbox is either overdetermined or two of the equations are not independent. Further, it can be easily proved that equations (1) to (4), and (7) to (9) are all independent equations and equations (5) and (6) can be derived from the other seven equations. By solving simultaneously the equations (1) to (4), and (7) to (9), the following expressions for the sizes of various gears in terms of $S$, $\phi$ and $a_1$ can be obtained.

$$\frac{a_2}{a_1}=\frac{\phi(S\phi^2-1)}{(S\phi^3-1)} \tag{10}$$

$$\frac{b_1}{a_1}=\frac{\phi^2}{[(\phi+1)-S\phi^2(\phi^2+1)]} \tag{11}$$

$$\frac{b_2}{a_1}=\frac{(S\phi^2-1)}{(S\phi^3-1)\,[(\phi+1)-S\phi^2(\phi^2+\phi+1)]} \tag{12}$$

$$\frac{b_3}{a_1}=\frac{\phi\,(\phi+1)\,(1-S\phi^2)}{[(\phi+1)-S\phi^2(\phi^2+\phi+1)]\,[(\phi+1)-S\phi^2(\phi^2+1)]} \tag{13}$$

$$\frac{c_1}{a_1}=\frac{1}{S\,\phi^2} \tag{14}$$

$$\frac{c_2}{a_1}=\frac{(S\phi^2-1)}{S\phi^2(S\phi^3-1)}$$

$$\frac{c_3}{a_1}=\frac{(\phi+1)(1-S\phi^2)}{S\phi^2[(\phi+1)-S\phi^2(\phi^2+1)]} \tag{16}$$

Though in the above equations $a_1$ has been taken as the reference gear, it is possible that at certain values of $S$ and $\phi$, some gears will be smaller than $a_1$. For each value of $S$ and this gear can be identified, and the sizes of the other gears can be expressed as multiples of

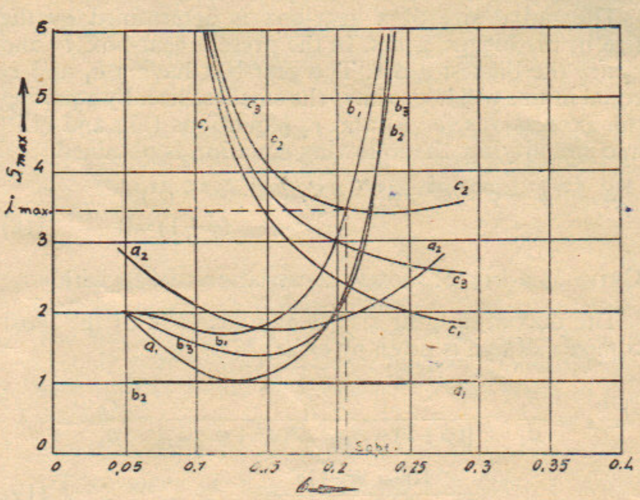



**Fig. 3**

**Variation of gear sizes with $S$ in 6S-8G-3Sh. Case 3 when $\phi=1.41$**

this. Figs. 2 and 3 show the variation with $S$ of the sizes of various gears, expressed as a ratio of the smallest gear, at $\phi=1.26$ and 1.41 respectively. It is noted that $a_1$ is the smallest gear for a greater range of $S$.

Since a gear cannot have negative size, both the numerator and denominator in equations (10) to (16) should be of the same sign for a given value of $\phi$. Equation (11) has the denominator positive for all values of $\phi$ and $S$. The denominator will be positive only when

$$\phi+1>S\phi^2\,(\phi^2+\phi+1)$$

*i.e.*

$$S_{\max}=\frac{\phi+1}{\phi^2\,(\phi^2+\phi+1)} \tag{17}$$

Curve 1 of Fig. 4, shows the variation of $S_{\max}$ with $\phi$.

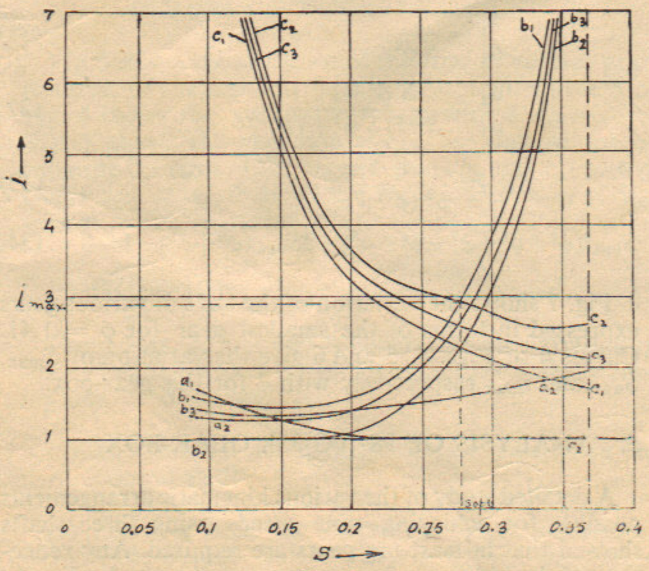



**Fig. 2**

**Variation of gear sizes with $S$ in 6S-8G-3Sh case when $\phi=1.26$**

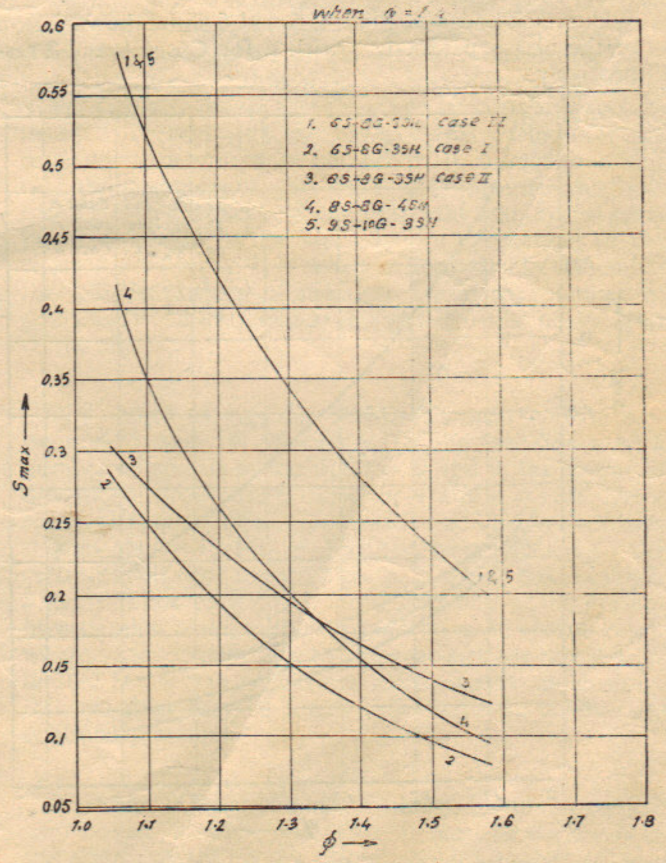



**Fig. 4**

**Variation of $S_{max}$ with $\phi$ in various gear boxes**

The radial size of a gear-box is determined by the size of the biggest gears. In the present gear-box, $b_1$ and $c_2$ are the biggest gears. The gear-box having $b_1$ and $c_2$ equal in size will hence have the smallest size. By equating the expressions for $b_1$ and $c_2$ [equations (11) and (15)] and simplifying, the following equation is obtained.

$$S_{opt}^2(\phi^7+\phi^6+\phi^5+\phi^4)-2S_{opt}(\phi^4+\phi^3+\phi^2)+(\phi+1)=0 \qquad (18)$$

Curve 1 of Fig. 5. shows the variation of $S_{opt}$ with $\phi$.

The maximum gear size in the gear-box of the most compact design is given by either $b_1$ or $c_2$.

$$i_{max}=\frac{b_1}{a_1}=\frac{\phi^2}{[(\phi+1)-S_{opt}\,\phi^2(\phi^2+\phi+1)]}=\frac{c_2}{a_1}$$

$$=\frac{(S_{opt}\,\phi^2-1)}{S_{opt}\,\phi^2\,(S_{opt}\,\phi^3-1)} \qquad (19)$$

Curve 1 of Fig. 6. shows the variation of $i_{max}$ with $\phi$.

Cases 1 and 2 have also been analysed and the following are the final results for $\phi=1.12$ and 1.26.

Case **1**:

$S_{max}=0\cdot235$ ; $0\cdot164$ (20)

$S_{opt}=0\cdot139$ ; $0\cdot103$ (21)

$i_{max}=5\cdot85$ ; $7\cdot15$ (22)

Case 2:

$S_{max}=0\cdot264$ ; $0\cdot207$ (23)

$S_{opt}=0\cdot152$ ; $0\cdot118$ (24)

$i_{max}=6\cdot27$; $7\cdot49$ (25)

Curves 2 and 3 of Figs. 4, 5 and 6 show the variation of the above parameters with $\phi$ for Cases 1 and 2 respectively.

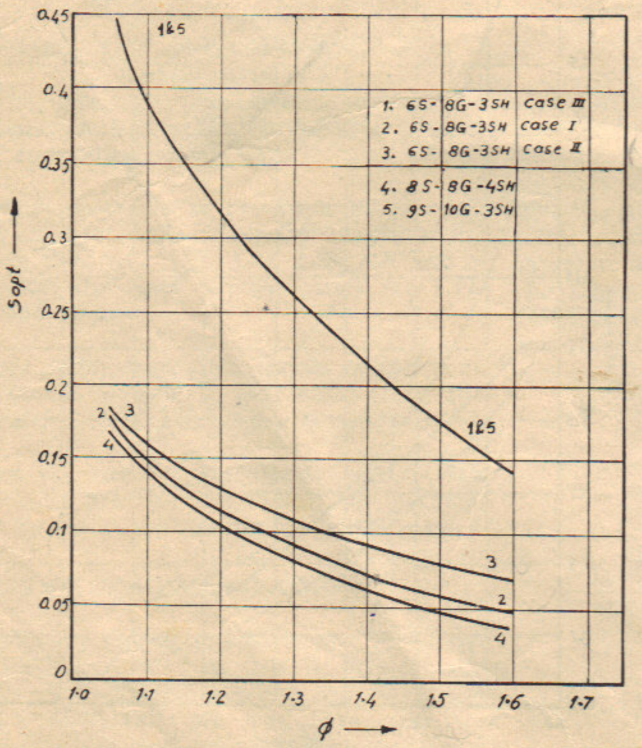



**Fig. 5**

**Variation of $S_{opt}$ with $\phi$ for various gear boxes**

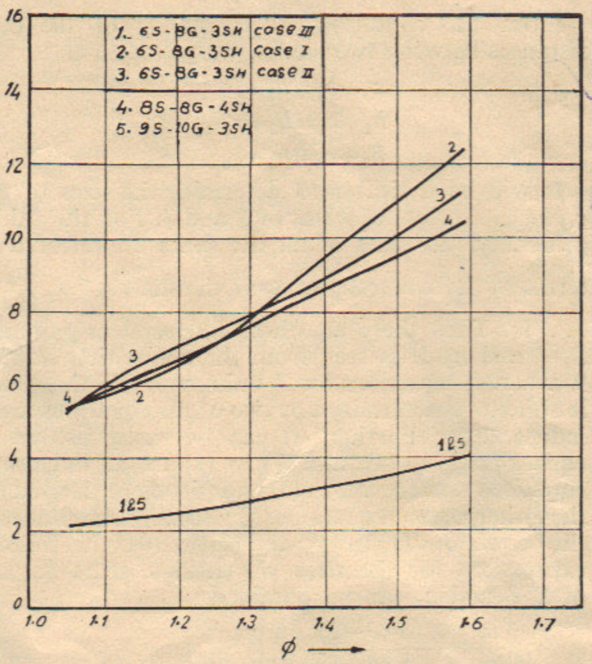



**Fig. 6**

**Variation of $i_{max}$ with $\phi$ for various gear boxes**

## 4. ANALYSIS OF 8S-8G-4Sh GEAR-BOX

The kinematic scheme and speed layout of 8S-8G-4Sh gear-box with minimum number of gears can be seen in Table 1. The analysis is similar to that of 6S-8G-3Sh gear-box. The following are the final results:

$$\frac{a_2}{a_1}=\frac{\phi^3(S\phi^2+1)}{(S\phi^5+1)} \qquad (26)$$

$$\frac{b_1}{a_1}=\frac{\phi\,(\phi^2+\phi+1)}{[1+S\,\phi^2(\phi^2+1)(\phi+1)]} \qquad (27)$$

$$\frac{b_2}{a_1}=\frac{(S\phi^2+1)(\phi^2+\phi+1)}{(S\phi^5+1)[1+S\,\phi^2(\phi^2+1)(\phi+1)]} \qquad (28)$$

$$\frac{c_1}{a_1}=\frac{(\phi^2+\phi+1)}{[1-S\,\phi^3(\phi+1)]} \qquad (29)$$

$$\frac{c_2}{a_1}=\frac{\phi(\phi^2+\phi+1)\,(S\,\phi^2+1)}{(S\,\phi^5+1)\,[1-S\,\phi^3(\phi+1)]} \qquad (30)$$

$$\frac{d_1}{a_1}=\frac{1}{S\,\phi^2} \qquad (31)$$

$$\frac{d_2}{a_1}=\frac{S\,\phi^2+1}{S\,\phi^2\,(S\,\phi^5+1)} \qquad (32)$$

$$S_{max}=\frac{1}{\phi^3\,(\phi+1)} \qquad (33)$$

$$S_{opt}^2(\phi^9+\phi^8+\phi^7+\phi^6+\phi^5)+2S_{opt}\,(\phi^4+\phi^3)-1=0 \qquad (34)$$

Fig. 7 shows the variation of the sizes of various gears expressed in terms of the smallest gear for $\phi=1.41$. Curves 4 of Figs. 4, 5 and 6 give the variation of $S_{max}$, $S_{opt}$ and $i_{max}$ respectively with $\phi$ for this gear-box.

## 5. ANALYSIS OF 9S-10G-3Sh GEAR-BOX

A detailed study of the various kinematic arrangements possible for achieving nine speeds using three shafts showed that at least ten gears are required. Any reduction of the number of gears was found to be leading to redundant equations.

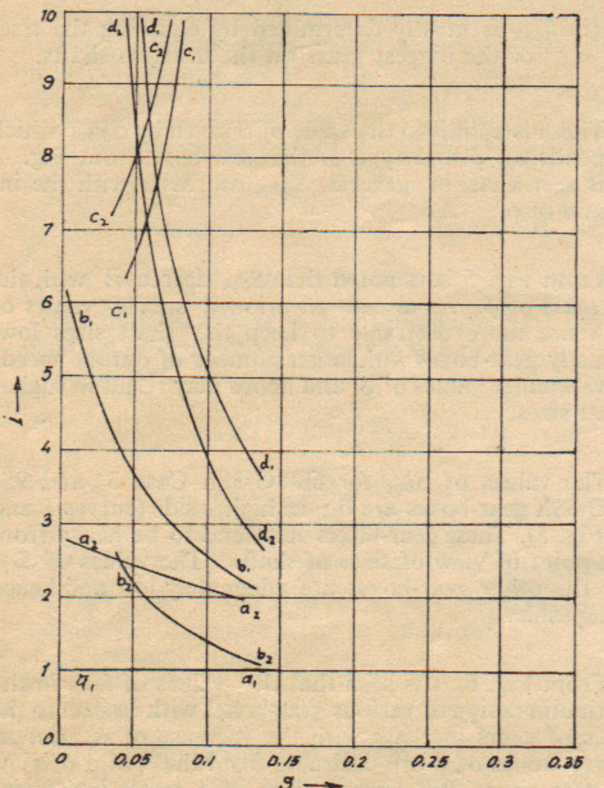



Fig. 7
Variation of gear sizes with $S$ in 8S-8G-4Sh- gear-box when $\phi$-1.41

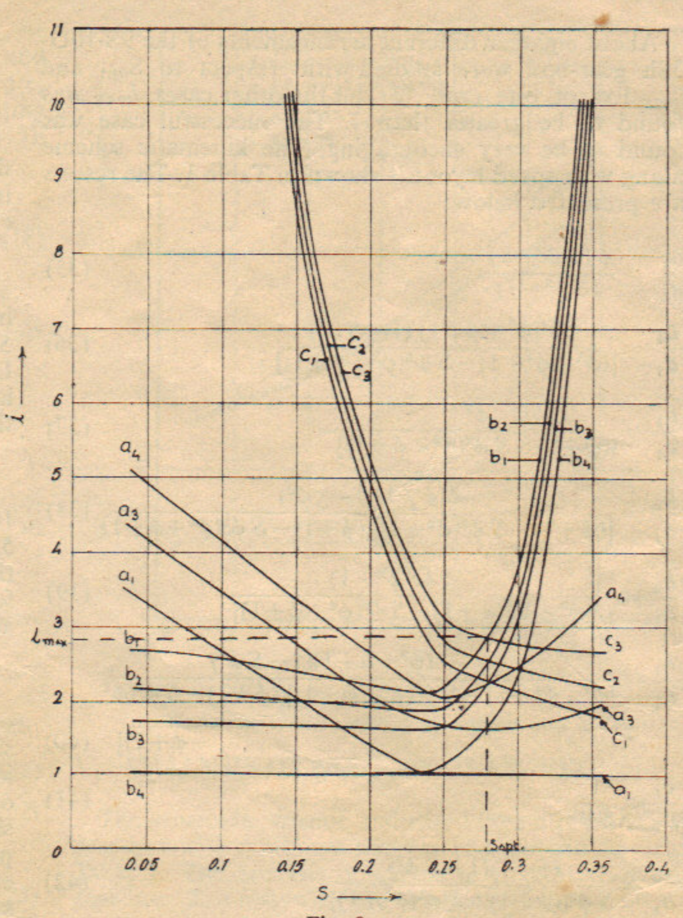



Fig. 8
Variation of gear sizes with $S$ in 9S-10G-3Sh gear box at $\phi = 1.26$

## 7. CONCLUSIONS

(i) 6S-8G-3Sh Case 3 and 9S-10G-3Sh gear-boxes reduce the number of gears by two in comparison with the conventional types without ad ding significant disadvantages. They are hence positively recommended for the gear-boxes with the number of output speeds of either 6 and 9 or multiples of 6 and 9.

(ii) 8S-8G-4Sh gear-box reduces the number of gears by 4 in comparison with the conventional type but at the cost of increased gear and shaft sizes. This has limited application and may be used where radial dimensions are not strictly limited but severe limitations on the axial dimensions exist.

(iii) Designers should try the suggested gear-boxes with reduced number of gears, before they resort to the conventional designs.

(iv) The number of speeds for which the gear-boxes are usually designed are multiples of 4, 6, 8 and 9. With the present analysis, the entire useful field of gear-boxes with minimum number of gears can be considered to have been analysed.

## 8. REFERENCES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