An Analysis of Forces in Oblique Cutting Based on the Lower Boundary of the Primary Deformation Zone

P. K. Venuvinod*, W. L. Jin** and C. M. Lee*

Abstract

Unlike the shear stress on the Merchant shear plane, the shear stress on the lower boundary of the primary deformation zone in cutting must be a material constant. Thus, modelling of orthogonal cutting based on the lower boundary has been particularly successful in predicting the power component of the cutting force. This paper extends this approach to oblique cutting. It is assumed that the progressive deformation of the work material into chip material occurs within the effective plane. An application of plasticity theory shows that certain simplifying assumptions concerning the three dimensional stress distribution on the lower boundary can be made with acceptable accuracy. The resulting model is capable of predicting the two main cutting force components lying in the cutting plane.

1. Introduction

A cutting operation in which the cutting edge deviates from the normal to the cutting edge by an angle of inclination, i, is said to be oblique. Chip formation and cutting mechanics in such operations tend to be three dimensional. Since a vast majority of cutting operations in practice are oblique to some degree, the prediction of cutting forces in such operations is a problem of practical significance.

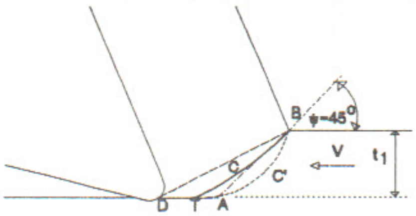
Classical models of oblique cutting [1,2] have been based on the utilisation of the notion of Merchant shear plane. In these models the prediction of any of the three cutting force components requires *a priori* knowledge of the mean shear stress, τ, on the shear plane and the apparent friction angle, λ, at the tool/chip interface and the chip thickness. Unfortunately, both τ and λ happen to depend on cutting conditions in addition to the tool/work material pair which often means that they cannot be known *a priori*. Thus, models based purely on the "Merchant" shear plane have been unable to *predict* any of the cutting force components even in the two dimensional case of orthogonal cutting (when i=0).

In contrast, the analysis of orthogonal cutting based on the lower boundary of the shear zone, as proposed by Rubenstein [3], has been able to predict the power component, f_c, of the cutting force merely from known magnitudes of the flow stress, s, of the work material and the chip thickness. This has been possible because the magnitude of s is a property of the work material prior to being subjected to any cutting deformation. Therefore, unlike τ, s must be a true material constant.

Previous attempts to extend the lower boundary concepts, which have proved highly successful in orthogonal cutting [3], to oblique cutting (for example, as in [4]), had relied on intuitive definitions concerning the geometry of and the stress distributions on the lower boundary. The present paper aims to present a more rigorous approach to extending the fundamental notions behind Rubenstein's model of orthogonal cutting [3] to the case of oblique cutting.

* City Polytechnic of Hong Kong

** Nanjing Aeronautical Institute



DB Merch. Shear Plane
DAB Assumed Path of Planes (AB : Pl. 1, DA : Pl. 2)
DTCB True Lower Boundary
DTC'B An Assumed Boundary Partly Overlapping the TLB

Fig. 1 LB of the Shear Zone in Orthogonal Cutting

2. The Lower Boundary of the Shear Zone in Orthogonal Cutting

Consider Fig. 1 which illustrates Rubenstein's model of orthogonal cutting [3]. Curve BD represents the true lower boundary (TLB) of the primary deformation zone whereas straight line BD represents the Merchant shear plane. Basing his arguments on the plasticity conditions near B, Rubenstein noted that, at B, the TLB must meet the unmachined surface at 45° and the normal stress on the TLB should be equal to the shear stress, s, which must be uniformly distributed over the TLB. Likewise, on the basis of the condition of chip/work material separation, it was noted that the TLB must be parallel to the cutting plane in the vicinity of D. Further, following a rigorous application of translational equilibrium criteria to the tool/chip/primary deformation zone/workpiece system, Rubenstein made the following observations: "Provided the stresses (normal stress, p, and shear stress, s) are uniformly distributed, the force components f_c (parallel to the cutting speed) and f_v (parallel to the normal to the machined surface) are *path independent*, i.e. along any boundary joining B and D, including the Merchant shear plane, the same force components will be obtained when the magnitudes of the stresses (p and s) are specified. Of the infinity of paths joining B and D, one of these is the TLB..., the force components acting on the TLB may be determined by calculating the force components along an arbitrarily chosen boundary joining the extremeties of the TLB provided (i) the stresses which are assumed to act on the arbitrarily chosen boundary are the same as those acting on the TLB, and (ii) the stresses are uniformly distributed".

Rubenstein next observed that if the assumed boundary (BC'TD) had a portion (TD) coincident with the TLB (BCTD) and the TLB is subjected, in its non-coincident part, to uniformly distributed stresses, the force components calculated by integrating over the assumed boundary will be the same as those calculated by integrating over the TLB, irrespective of whether or not the assumed boundary has any physical significance (see Fig. 1).

Based on this premise, Rubenstein chose an assumed boundary represented by planes BA and AD which are tangential to the TLB at the latter's extremities B and D respectively. Following the above arguments, (i) the shear stress on BA and AD were taken to be uniform and equal to s, (ii) the normal stress on BA was taken to be uniformly distributed and equal to s, and (iii) the normal stress on AD was taken to be of unknown distribution and magnitude. Integration of these stresses across path BAD yielded an expression for f_c which was a function of just s, the uncut area and the Merchant shear angle. This expression has since been shown to be quite accurate and robust in the light of extensive experimental data obtained in orthogonal cutting conditions.

3. The TLB in Oblique Cutting

We now examine how the basic notions contained in Rubenstein's model of TLB in orthogonal cutting [3] may be extended to single edge oblique cutting. Fig. 2 illustrates the proposed approach. Plane B'B"D"D' is the Merchant shear plane. The TLB is a curved surface joining B'B" and D'D". As in [3], the TLB is assumed to (i) be tangential to the cutting plane in the vicinity of the cutting edge and (ii) meet the unmachined surface at the other end, B'B", at an angle ψ when measured in the plane normal to the cutting edge. The exact geometry of the TLB between the two extremeties (B'B" and D'D") is ofcourse unknown. For the purpose of identifying the direction of shear on the TLB, it is assumed that the progressive transformation of the velocity **V** of the uncut work material into velocity $\mathbf{V}_c$ of the chip material takes place within the effective plane (EP) i.e. within the plane containing vectors **V** and $\mathbf{V}_c$ (and, therefore, the shear velocity vector, $\mathbf{V}_s$, on the Merchant shear plane). Thus, the curve of intersection DTB between the TLB and the effective plane may be assumed to represent the direction of shear along the TLB.

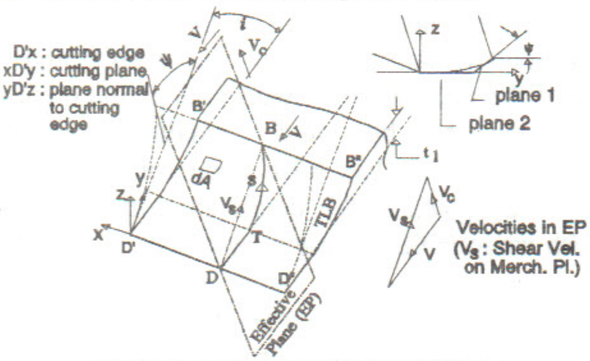


Fig. 2 LB of the Shear Zone in Oblique Cutting

The authors have recently undertaken a 3D stress analysis of the above configuration assuming that (i) the work material is a perfectly plastic and isotropic continuum, and (ii) line B'B" is parallel to the cutting edge [5]. The analysis is too lengthy to be presented in this brief paper. However, the analysis has shown that the following conclusions may be drawn within acceptable accuracy (within 2% error):

(i) At the free surface (i.e. at B'B") the TLB is inclined to the surface of the uncut workpiece at 45° , i.e. $\psi = 45°$.

(ii) The shear stress along the TLB at B'B" occurs in a direction parallel to the velocity of shear along the TLB at B'B".

(iii) The magnitude of the normal stress, p, on the TLB at B is equal to $s.\cos\eta_{sB}$, where η_{sB} is the angular deviation of the shear vector on the TLB at B from the normal plane when measured in a plane tangential to the TLB at B.

The following further assumptions are motivated by arguments similar to those presented in [3] for the case of orthogonal cutting :

(i) The shear stress over the TLB is uniformly distributed and is directed along the curve DTB.

(i) In the vicinity of the cutting edge, a portion, T'T"D'D", of the TLB is parallel to the cutting plane. The normal stress, p_m, on this portion is of unknown magnitude.

(ii) The normal stress on the portion B'B"T"T' of the TLB, which is not parallel to the cutting plane, is uniformly distributed and, by virtue of conclusion iii above, has its magnitude equal to $s.\cos\eta_{sB}$. These stresses need to be integrated over the TLB in order to estimate the cutting forces. However, this integration requires *complete* knowledge of the geometry of the TLB which, unfortunately, is not available. A similar problem was faced by Rubenstein in the case of orthogonal cutting which prompted him to utilise the notion of *path independence* in [3]. We will explore the feasibility of such a notion in the context of oblique cutting in the next section.

4. Force Prediction in Oblique Cutting

Consider an area element of area dA located on the TLB. Let $\mathbf{u}_{dA}$ be the unit vector normal to the area element. Let s and p_{dA} be the shear and normal stresses on dA. Clearly, p_{dA} acts in the direction of $\mathbf{u}_{dA}$ so that $p_{dA}(dA)\mathbf{u}_{dA}$ is the vector representing the normal force on dA. The shear stress however is *assumed* to be directed along the line of intersection of the effective plane with the area element. Thus the shear force vector on dA is given by $s(dA)\{(\mathbf{u}_{dA} x \mathbf{u}_{EP})/\sin\theta_{dA}\}$ where $\mathbf{u}_{EP}$ is the unit vector normal to the effective plane, "x" represents the vector product operation and θ_{dA} is the angle between $\mathbf{u}_{EP}$ and $\mathbf{u}_{dA}$. We now desire to find the contribution, $d\mathbf{f}_j$, of the normal and shear forces arising from dA to the total force $\mathbf{f}_j$ in the direction of an arbitrary unit direction vector $\mathbf{u}_j$. Clearly this can be achieved using the scalar product operation (*) as follows :

$$df_j = \{s(dA)(\mathbf{u}_{dA} x \mathbf{u}_{EP})/\sin\theta_{dA}\}^* \mathbf{u}_j + p_{dA}(dA)\mathbf{u}_{dA}{}^* \mathbf{u}_j$$

$$= s\{(d\mathbf{A}/\sin\theta_{dA})x(\mathbf{u}_{EP})\}^* \mathbf{u}_j + p_{dA}(d\mathbf{A}^* \mathbf{u}_j) \quad (1)$$

where d**A** is the area vector of the area element dA.

The total force $\mathbf{f}_j$ is obtained by integrating $d\mathbf{f}_j$ over the entire area of the TLB as

$$\mathbf{f}_j = [\int_{TLB}\{s(d\mathbf{A}/\sin\theta_{dA})x\mathbf{u}_{EP} + p_{dA}d\mathbf{A}\}]^* \mathbf{u}_j \quad (2)$$

The above equation is still inadequate for estimating $\mathbf{f}_j$ since the geometry of the TLB which, strictly speaking, ought to be the path of integration is unknown. We will now explore the possibility of using an assumed path of integration. One candidate path consisting of two planes, namely planes 1 and 2, similar to the method adopted in orthogonal cutting [3] is illustrated in Fig. 1. In order to enable force estimation from such an assumed path, equation now can be rewritten as

$$\mathbf{f}_j \approx [\Sigma_{k=1,2}\{s(\mathbf{A}_k/\sin\theta_k)x\mathbf{u}_{EP} + p\mathbf{A}_k\}]^* \mathbf{u}_j \quad (3)$$

where $\mathbf{A}_k$ is the area vector of and p_k the magnitude of the normal stress on plane k in the assumed path and θ_k is the corresponding value of θ.

Note that since the bounding edges B'B" and D'D" of this path are common with the bounding edges of the Merchant shear plane, B'B"D"D', the sum, $\Sigma k=_{1,2}\mathbf{A}_k$, of the area vectors of planes 1 and 2 must equal the area vector $\mathbf{A}_m$ of the Merchant shear plane, i.e. $\Sigma_{k=1,2}\mathbf{A}_k = \mathbf{A}_m$. Thus, but for the presence of the term "$\sin\theta_k$", equation 3 should yield a path independent estimate of $\mathbf{f}_j$. We will now explore the deviation from path independence introduced by the term $\sin\theta_k$.

Consider now the estimation of a force component which is parallel to the cutting plane from equation 3. The area vectors, $\mathbf{A}_k$, are easily determined from the chip geometry. The value of s is assumed to be known *a priori*. Following the conclusions described in the previous section, the magnitude of p on plane 1 may be taken equal to $s.\cos\eta_{sB}$. Since the desired force is parallel to the cutting plane, the unknown normal stress p_m on plane 2 has no effect on the desired force. (Note that force f_v cannot be estimated by equation 3, just as in the case of orthogonal cutting [3], since its magnitude is a function of the unknown p_m). Thus, for a given path, equation 3 can provide estimates of the force components f_c (parallel to the cutting speed vector, **V**) and f_l (parallel to the normal to **V** in the cutting plane) simply from a knowledge of s, uncut area and chip geometry. A numerical evaluation of f_c and f_l from equation 3 for the range of paths obtained by varying ψ in the range 0 to 80° has shown that the estimates remain fairly constant. The theoretical maximum error has been found to be of the order of 10% while the actual error is expected to be much smaller. More interestingly, it has been been found that the term $\Sigma_{k=1,2}(\mathbf{A}_k/\sin\theta_k)$ could be approximated with reasonable accuracy by $(\mathbf{A}_m/\theta_m)$. Thus,

$$\mathbf{f}_j \approx s\{(\mathbf{A}_m/\sin\theta_m)x\mathbf{u}_{EP} + \cos\eta_{sB}\mathbf{A}_m\}^* \mathbf{u}_j$$
: for direction j in the cutting plane (4)

It must be reiterated that although the force estimates from equation 4 are only approximate, the errors introduced are within the acceptable range for the practical purpose of estimating force components parallel to the cutting plane. Interestingly, when obliquity i=0, equation 4 yields the path independent solution presented for f_c in [3]. Future research may be directed to the identification of a truely *path independent* three dimensional solution to the problem - if such a solution does indeed exist in the case of oblique cutting.

5. Experimental Verification of the New Model

The predictive ability of equation 4 has been tested against previously published experimental data on single edge oblique cutting of an aluminium alloy, copper and mild steel [6] and further experimental data obtained by the present authors while cutting another aluminium alloy [5]. The experimental procedure used in the further experiments was identical to that adopted in [6]. Chip thicknesses, chip flow angles and cutting forces were measured while dry cutting with a range of cutting tools (i = 0 to 50°, normal rake angle=25 and 30°, t_1=0.05-0.175 mm). Edge forces were removed from the measured force components by the method used in [6] for yielding the magnitudes of f_c and f_l. The calculation method is summarised in the Appendix.

Fig. 3 shows the observed relationship between the coeffcient of s in equation 4 and

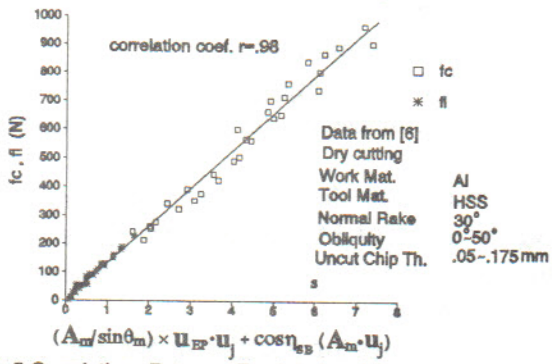


Fig.3 Correlations Between Predicted and Measured Forces

the experimentally obtained forces for the case of machining aluminium alloy reported in [5]. It is clear from the large magnitude of the linear correlation coefficient (overall r=0.98) that the model exhibits a high degree of predictive ability with regard to both f_c and f_l while machining aluminium alloy under the range of cutting conditions covered by Fig. 3.

Table 1 summarises the results obtained

Tab.1 Results obtained by applying eq(4)

Wk. Mat.	Norm. Rake	f_c		f_l		overall		[ ]
		s	r	s	r	s	r	
Al	30°	236	.98	203	.99	215	.98	5
	25°	237	.96	234	.98			
	30°	135	.96	118	.98	132	.98	6
Cu	30°	249	.92	189	.98	249	.91	6
MS	20°	355	.93	253	.95	348	.98	6

where s: MPa; r: correlation coefficient

from the four sets of test data. Note the magnitudes of s derived from f_c data. These magnitudes, which are for i>0, are close to the corresponding magnitudes of s derived from f_c data obtained while orthogonally cutting (i.e with i=0) the same work material. It may be concluded on the basis of this observation that the predictive power of the new model with respect to f_c is good. However, the result is not as good when a similar criterion is applied to prediction with respect to f_l. In particular, while machining copper or mild steel there are significant mismatches between the magnitudes of s derived from f_c and f_l data.

A probable reason for the poorer performance of the model with regard to f_l is that the assumption of uniformly distributed normal stress, p, might not be strictly valid over the portion B'B"T"T' of the TLB which is non-coincident with the cutting plane. This aspect has been examined in detail in [5] where the normal stress on the non-coincident portion of the TLB was taken to be a fraction C of $s.\cos\eta_{sB}$ (equation 4 assumes that C=1). It was demonstrated that, for each combination of work material and normal rake angle, one can determine a unique value of C such that there is complete agreement between the magnitudes of s derived from f_c and f_l data. In particular it was shown that the magnitude of C for aluminium was higher than that for mild steel which, in turn, was higher than that for copper. Thus, provided the chip formation geometry and the magnitudes of s and C for the work material are known, the model based on TLB can be used to predict the magnitudes of both f_c and f_l with a high degree of confidence.

In the above, we have presented a new model of oblique cutting which is based on a rigorous analysis of the TLB and is capable of predicting cutting force components parallel to the cutting plane in oblique cutting. As noted earlier, models based purely on the Merchant shear plane, such as in [1,2], have no ability to predict any of the cutting force components. The model used in [6], although capable of predicting both f_c and f_l, requires that not only the flow stress, s, on the TLB but also the shear stress, τ, on the Merchant shear plane be taken as a material constant. The requirement to consider τ as a material constant diminishes the elegance of this approach. The model developed in [4], which was also based on an analysis of the TLB in oblique cutting, utilised an intuitively defined path for stress integration and lacks the rigour underlying the present model.

6. Conclusion

A new analysis of single edge oblique cutting based on a model of the geometry of and stress distributions on the true lower boundary of the primary deformation zone has been presented above. The model is an improvement over previous attempts aimed at extending Rubenstein's approach for modelling orthogonal cutting [4] to the case of oblique cutting. Equation 4 may be utilised to predict the force component f_c with reasonable accuracy from a knowledge of the flow stress, s, the TLB of the work material, the uncut area, the chip thickness and the chipflow angle. However, in order to predict both f_c and f_l with reasonable accuracy, equation 4 needs to be modified to include a constant C which charactarises the degree of non-uniformity of the normal stress distribution on the TLB.

Acknowledgement

The authors wish to thank the City Polytechnic of Hong Kong for the financial support provided to this work.

References

[1] M.E. Merchant, *J. appl. Mech.* **66**,A168 (1944).

[2] M.C.Shaw, N.H. Cook and P.A. Smith,*Trans. Am. Soc. mech. Engrs* **74**,1055 (1952).

[3] C. Rubenstein, *Int. J. Mach. Tool Des. Res.* **12**, 121 (1972).

[4] P.K. Venuvinod and W.S. Lau, Trans. *Am. Soc. mech. Engrs, J. Eng. Ind.* **100**, 287 (1978).

[5] P. K. Venuvinod, C.M. Lee and W. L. Jin, Prediction of Forces in Oblique Cutting, *ME Research Bulletin*, City Polytechnic of HK **1** (1992).

[6] W.S. Lau and C. Rubenstein, *Int. J. Mach. Tool Des. Res.* **23**, 21 (1983).

Appendix

<u>Given</u>

t_1 = uncut chip thickness
t_2 = chip thickness
b = workpiece width
α_n = normal rake angle
i = angle of obliquity
η_c = chip flow angle
s = flow stress of the uncut work material

<u>Let</u>

ϕ_n = normal shear angle (Merchant)
A_m = Area of Merchant shear plane
$\mathbf{u}_V$ = unit vector parallel to the cutting velocity
$\mathbf{u}_{Vc}$ = unit vector parallel to the chp velocity
$\mathbf{u}_m$ = unit vector normal to the Merchant shear plane
$\mathbf{u}_{fc}$ = unit vector along f_c
$\mathbf{u}_{fl}$ = unit vector along f_l

<u>Then</u> to compute f_c and f_l from equation 4 use the following procedure :

$$\phi_n = \arctan\{\cos\alpha_n/(t_2/t_1 - \sin\alpha_n)\} \quad (A1)$$
$$A_m = bt_1/(\cos i.\sin\phi_n) \quad (A2)$$
$$\eta_{sB} = \arctan[\{\tan i.\cos(\pi/4-\alpha_n)-\tan\eta_c\sin(\pi/4)\}/\cos\alpha_n] \quad (A3)$$
$$p = s.\ \cos\eta_{sB} \quad (A4)$$

Compute the direction cosines of various unit vecors along axes (x, y, z) respectively in Fig. 2 as follows:

$$\mathbf{u}_V : (\sin i, -\cos i, 0) \quad (A5)$$
$$\mathbf{u}_{Vc} : (\sin\eta_c, -\cos\eta_c\sin\alpha_n, \cos\eta_c\cos\alpha_n) \quad (A6)$$
$$\mathbf{u}_m : (0, \sin\phi_n, -\cos\phi_n) \quad (A7)$$
$$\mathbf{u}_{fc} = \mathbf{u}_V \quad (A8)$$
$$\mathbf{u}_{fl} : (-\cos i, -\sin i, 0) \quad (A9)$$
$$\mathbf{A}_m = A_m.\mathbf{u}_m \quad (A10)$$
$$\mathbf{u}_{EP} = (\mathbf{u}_V x \mathbf{u}_{Vc})/\sin\{\arccos(\mathbf{u}_V * \mathbf{u}_{Vc})\} \quad (A11)$$

For computing f_c, take $\mathbf{u}_j=\mathbf{u}_{fc}$ in equation 4.
For computing f_l, take $\mathbf{u}_j=\mathbf{u}_{fl}$ in equation 4.