An Analysis of Symmetrical V-Cutting

P. K. Venuvinod, W. S. Lau, C. Rubenstein (1)

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Past analyses of symmetrical V-cutting are re-examined. These led to the conclusion that the process could not be explained in terms of a model based on twin shear planes (one at each edge) - instead it was suggested that the process could be analysed in terms of an "overall" shear plane which did not contain the cutting edges! It is shown here that insufficient regard has been paid to the "interaction" between the chip elements originating at each of the cutting edges and this has led to conceptual inconsistencies. Taking into account the consequencies of this interaction, an explanation of symmetrical V-cutting based on a twin shear plane model is offered. Deductions from the model are shown to be consistent with observation.

INTRODUCTION

What insight we have into vee cutting can be dated from Zorev's examination [1] of a particular simple case of double edge cutting. Later, Armarego [2] and Luk [3] studied orthogonal symmetric vee-cutting, the former subsequently extending his investigation to include oblique vee-cutting [4]. The results in [2] were explained in terms of an overall shear plane (which could not, of course, include the individual edges) but in [4] the concept of twin shear planes passing through each cutting edge was examined. However, several inconsistencies emerged and, in consequence, it was decided to abandon the twin shear plane concept in favour of an overall shear plane model. This conclusion was reached, in our opinion, because the interaction which must occur between the chip segments originating at each of the cutting edges was ignored.

The aim of the present paper is to propose a model of symmetric vee-cutting in which the twin shear plane theory is resuscitated and the interaction between the two chip segments is taken into account.

THEORY

Geometric and Kinematic Considerations:

Fig. 1 shows a schematic diagram of chip formation in symmetric vee-cutting. This is based on the twin shear plane concept and illustrates the case when the "overall obliquity" of the process is zero, i.e. when the line of intersection of the unmachined work surface with the tool rake plane (AB) is perpendicular to the cutting velocity vector, $\bar{V}$.

The tool has a back rake of α_b and a semi-vee angle, δ', in the rake plane (OAB) corresponding to the semi-vee angle, δ, of the uncut chip section O'A'B'. Obviously, when $\alpha_b \neq 0$, the cutting edges OA and OB are not perpendicular to the cutting velocity vector, $\bar{V}$, and will present some obliquity (say i). Likewise, when $\alpha_b \neq 0$, the normal rake angle, α_n, at each cutting edge will not be equal to α_b. The chip flow angle, η_c, at each cutting edge will obviously be equal to $90^\circ - \delta'$.

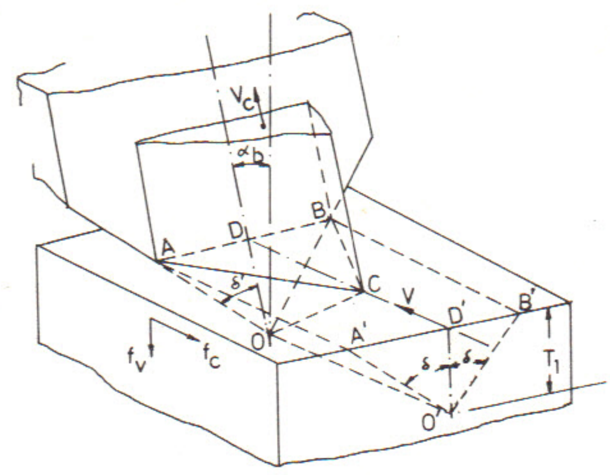

Fig. 1 Schematic Representation of Symmetric Vee Form Cutting

The vee-depth of the symmetric triangular uncut chip section, O'A'B', is T_1. Let A_1 be its area ($= T_1^2 \tan\delta$). If chip distortion is negligible, the chip cross-section will also be triangular with the same base width A'B' (=AB) but with a vee-depth, T_2, different from T_1. The cross-sectional area, A_2, of the chip is $T_1T_2 \tan\delta$.

OAC and OBC are the triangular shear planes originating from the cutting edges, OA and OB. Let ϕ_n be the normal shear angle associated with each shear plane. Since the cutting velocity, V, as well as the chip velocity V_c, are identical at each cutting edge, the shear velocities, V_s, along the two shear planes will also be equal. However, since neither $\bar{V}$ nor $\bar{V}_c$ is perpendicular to the cutting edge when $\alpha_b \neq 0$, the shear velocity vector, $\bar{V}_s$, in general, will deviate by an angle η_s from the normal to the cutting edge in the shear plane.

Expressions for i, α_n, δ', η_c, ϕ_n and η_s in terms of α_b, δ, and the ratio T_2/T_1 are available from [2,4].

Force Considerations on Each Chip Segment:

Fig. 2 shows the chip formed in symmetric vee-cutting after it has been partitioned into two equal chip segments (1 and 2) by a partition plane which is normal to the rake plane and passes through the line of symmetry (OZ) of the tool rake profile (AOB).

Consider now chip segment 1 as a rigid body bounded by the shear plane OAC, the rake plane and the partition plane. As in conventional single edge cutting, this chip segment will experience the shear and normal forces, S and N_s, at the shear plane OAC and half of the tangential and normal forces, F and N, acting at the chip/rake interface.

In the absence of chip segment 2, the cutting process at edge 1 (OA) would be the result of conventional oblique cutting and the chip would exit at a chip flow angle and chip velocity, say η_{co} and V_{co}, appropriate to the process characteristics of free oblique cutting. However, due to the presence of chip segment 2, chip segment 1 must experience some reaction force, F_{in}, as a result of which it assumes the common chip flow angle $\eta_c = 90^\circ - \delta'$ and a common chip velocity, V_c (not, in general, equal to V_{co}). Likewise, chip segment 2 must experience a reaction force equal but opposite to F_{in}.

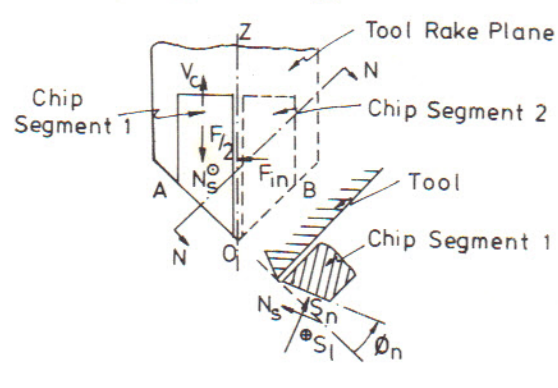

Fig. 2 Forces on Chip Segment 1

F_{in} is normal to the partition plane and thus acts parallel to the rake plane and normal to the common chip velocity vector, $\bar{V}_c$. When chip segments 1 and 2 are combined and viewed as an entity, force F_{in} is balanced within the chip (hence it has been designated as an internal force, F_{in}). However, from the point of view of each chip segment, viewed as a separate rigid body, F_{in} is an external force.

It is clear from the above that the existence of force F_{in} represents the one feature that distinguishes the mechanics of chip formation at each cutting edge in vee-cutting from that occurring in conventional single edge free cutting. We shall examine in the next section the implications of superposing F_{in} onto a conventional single edge free cutting process.

Superposition of F_{in}:

Consider Fig. 3a which illustrates a conventional single edge free cutting process with an obliquity i. Let the chip flow angle and the chip velocity in this state (state 0) be equal to η_{co} and V_{co} respectively. Since the relative velocity between the chip and the tool is in the direction of V_{co}, we can expect the tangential force (frictional drag) between the chip and tool rake, F_o, to be collinear with (but opposite in direction to) V_{co}. The angular deviation, η_{Fo}, of this force F_o from the normal to the cutting edge is obviously equal to η_{co}, i.e. $\eta_{Fo} = \eta_{co}$.

Now let a constant external force F_{in} be applied to the chip in a direction perpendicular to the chip flow velocity vector, $\bar{V}_{co}$, and parallel to the rake face. As a result, the chip will move from its original state (state 0) to a new state (state 1 in Fig. 3b). We will assume that vector $\bar{F}_{in}$ of constant magnitude, F_{in}, has remained perpendicular to the instantaneous chip velocity vector throughout the transition of the chip from state 0 to state 1. During this transition, the force configuration on the chip will, of course, be continuously changing, until the chip finally reaches the equilibrium state 1. Let η_c, V_c and F be the chip flow angle, chip velocity and chip/tool frictional drag respectively in this equilibrium state 1.

In contrast to the initial, 0, state there are now two forces acting on the chip, these being parallel to the rake face. The resultant force tangential to the rake plane is now F' given by $\bar{F}' = \bar{F}_{in} + \bar{F}$. The angular deviation of this total force parallel to the rake plane, F', from the normal to the cutting edge, η_F, (see Fig. 3b) is given by

$$\eta_F = \eta_c - \arctan(F_{in}/F) \qquad (1)$$

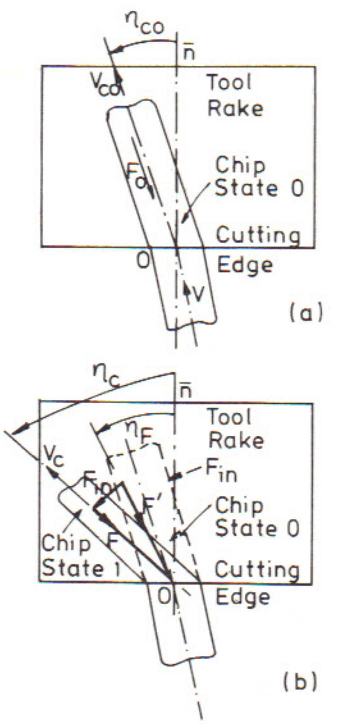

Fig. 3 (a) Chip State in the absence of F_{in}
(b) Chip State with the super position of F_{in}

For finite magnitudes of F_{in} and F, $\eta_F < \eta_o$ i.e. the direction of the force F moves closer to the normal plane. Further, since for chip equilibrium, F' must be balanced by an equal force acting on the shear plane, that balancing force also moves towards the normal plane.

This means that *from the force configuration viewpoint*, the superposition of F_{in} makes any oblique cutting process less oblique, i.e. "more orthogonal". As $F_{in}/F \rightarrow \tan\eta_c$, $\eta_F \rightarrow 0$ and we have a situation where the process obliquity is effectively zero (once more only from the viewpoint of cutting forces because neither the cutting velocity, V, nor the chip velocity, V_c, is orthogonal to the cutting edge). Such a situation is not unprecedented, for, when machining with self propelled rotary tools [5,6], the cutting force configuration is orthogonal although the cutting velocity, V, and the chip velocity, V_c, lie out of the normal plane. The condition when $\eta_F = 0$ will be described as "pseudo-orthogonal".

To review the discussion so far : we have seen that we cannot hope to explain vee-cutting solely in terms of what we have learnt from single edge free oblique cutting. Whatever insight we gain from this source must be supplemented by a knowledge of the effects of F_{in} - one of which we have seen already viz., a modification of the cutting process *towards* cutting force orthogonality.

Estimation of F_{in} and other Force Components:

It is possible to obtain an estimate of F_{in} in terms of the reduced cutting and thrust force components, f_c in the direction of cutting speed and f_v in the direction normal to the unmachined work surface, by applying the principles that

(i) each chip segment must be in equilibrium i.e. the vector sum of all forces acting on it (namely S, N_s, F/2, N/2 and F_{in}) is equal to zero; and

(ii) for each chip segment, the shear force vector, $\bar{S}$, at the shear plane is collinear with the shear velocity vector, $\bar{V}_s$, and the tangential force at the chip/rake interface ($\bar{F}$) is collinear with the chip velocity vector, $\bar{V}_c$.

The resulting expression is

$$F_{in} = \frac{F\{\cos\delta' + \sin\delta' \sin\nu_n \tan\eta_s\} - N\cos\nu_n \tan\eta_s}{2\{\sin\delta' - \cos\delta' \sin\nu_n \tan\eta_s\}} \qquad (2)$$

where

$$\nu_n = \phi_n - \alpha_n \qquad (3)$$

$$F = f_c \sin\alpha_b + f_v \cos\alpha_b \qquad (4)$$

and

$$N = f_c \cos\alpha_b - f_v \sin\alpha_b \qquad (5)$$

While the principle of collinearity has been invoked in the derivation of F_{in}, we might note that there are numerous occasions when empirical data are seriously at variance with this principle. If, on this basis, the assumption of collinearity were to be abandoned, it would be necessary to introduce some alternative assumption in order to obtain an expression for F_{in}. While the form of the assumption would influence the expression for F_{in}, *it could have no influence on the concept of F_{in} which is fundamental.*

Once the magnitude of F_{in} is obtained, the magnitudes of the other force components acting on each chip segment can be readily obtained from the principle of chip equilibrium as follows :

$$S_\ell = (F/2)\cos\delta' - F_{in}\sin\delta' \qquad (6)$$

$$S_n = (N/2)\cos\nu_n - (F/2)\sin\delta'\sin\nu_n - F_{in}\cos\delta'\sin\nu_n \qquad (7)$$

and

$$N_s = (N/2)\sin\nu_n + (F/2)\sin\delta'\cos\nu_n + F_{in}\cos\delta'\cos\nu_n \qquad (8)$$

where S_ℓ and S_n are the components of the shear force S along and normal to the cutting edge respectively.

EXPERIMENTS AND RESULTS

Dry symmetric vee form cutting tests were performed at a cutting speed, V, equal to 1 m/min on flat aluminium alloy workpieces mounted on a piezo-electric dynamometer. Twenty different cutting tools of M2 H.S.S. material were ground with different combinations of δ and α_b. Chip length, ℓ_2, cutting force component in the direction of cutting speed, F_c, and thrust force component in the direction normal to the unmachined work surface, F_V, were measured during cutting tests at different uncut vee-depths, T_1, with each tool.

From volume consistency between the materials approaching and leaving the cutting zone

$$\ell_1 A_1 = \ell_2 A_2 \quad (9)$$

where ℓ_1 is the length of workpiece in the direction of cutting speed, V; $A_1 = T^2_1 \tan\delta$ and $A_2 = T_1 T_2 \tan\delta$. Using these relationships, the chip thickness ratio in terms of vee-depths, T_2/T_1, was obtained from the measured value of chip length, ℓ_2, as $T_2/T_1 = \ell_1/\ell_2$.

For each tool, ratios F_c/T_1 and F_V/T_1 were then plotted against T_1. The resulting graphs were linear as in earlier works [2,4] so that following [2,4] the slopes of these lines gave the magnitudes of f_c/T_1^2 and f_v/T_1^2 for each tool, f_c and f_V being the components of F_c and F_V respectively which are directly associated with chip formation.

Table 1 summarises the results obtained from these symmetric vee form cutting tests on aluminium alloy work material. The observed variations of T_2/T_1, f_c/T_1^2 and f_v/T_1^2 with δ and α_b are similar to those reported by Armarego [2].

To afford a comparison between symmetric vee-cutting and single edge free cutting, a separate series of dry single edge free oblique tests using the same tool/work material combination and cutting speed was performed. These tests covered a wide range of edge obliquities ($i = 0$ to 50^o), normal rake angles ($\alpha_n = 0$ to 30^o) and uncut chip thicknesses ($t_1 = 0.1$ to 0.5 mm). The data so obtained were analysed according to the procedures described in [7]. The resulting behaviour of the various cutting parameters was essentially similar to that reported in [7] and, hence, is not presented here.

However, no direct correlation between the data in single edge free cutting and symmetric vee-cutting was evident when the magnitudes of ϕ_n, f_c and f_v in the one process were compared with those in the other for identical combinations of i, α_n and A_1. This is not surprising since identity could have been expected only if there were no interaction between chip segments 1 and 2 in vee-cutting. Having thus established the importance of F_{in}, the data in Table 1 and those reported in [2] were further processed using equations (1 to 8). Some of the results so obtained are discussed next.

DISCUSSION

Consider the deduction that, as a result of the existence of the internal force F_{in}, the force configuration in vee-cutting tends towards the normal plane although velocity vectors, $\bar{V}$ and $\bar{V}_c$, may deviate significantly from this plane. To test this deduction, equation (1) has been used to analyse the data presented in Table 1. The resulting values of η_F are quoted in the table whence it can be seen that $-10^o < \eta_F \leqslant 11^o$. In contrast, the magnitudes of i and η_c (representing the angular deviations of vectors, $\bar{V}$ and $\bar{V}_c$, from the normal plane) are found to vary over a much wider range, viz. $0^o < i < 36^o$ and $0^o < \eta_c < 67^o$. Similarly, when data given in [2] were analysed, it was observed that the ranges of the variables η_F, i and η_c were $3^o < \eta_F < 13^o$, $0 < i < 18^o$ and $0 < \eta_c < 62^o$ respectively. For comparison, consider data obtained in free oblique cutting of nominally the same aluminium alloy. In [8] it was shown that when the chip flow direction η_c was 45^o, $\eta_F \simeq 42^o$ - in contrast the present experiments show that in vee cutting for $\eta_c = 45^o$, $\eta_F = 9.9^o$ (Table 1). Clearly then, as predicted, the force configuration in symmetric vee-cutting tends towards the normal plane more than does the configuration in single edge free oblique cutting.

Notwithstanding the pronounced deviation of the chip flow direction from the normal plane (i.e. as much as 67^o), the values of η_F are surprisingly small - for both Armarego's data [2] and our own, $-10^o < \eta_F < 13^o$. Thus *as an approximation*, we may class these experimental conditions as pseudo-orthogonal from the view-point of force configuration. On this basis we might examinine the extent to which expressions for reduced cutting and thrust force components, f_c and f_v, derived for conventional *free orthogonal* cutting are applicable.

With this objective, we will examine two results from Connolly and Rubenstein's analysis of free orthogonal cutting [8] viz.

$$f_c = sA_1(\cot\phi_n + 1) \quad (10)$$

$$f_v = pA_1(\cot\phi_n - 1) \quad (11)$$

TABLE 1

Experimental Data in Vee-form Cutting

δ (deg)	α_b (deg)	T_2/T_1	f_c/T_1^2 (MPa)	f_v/T_1^2 (MPa)	η_F (deg)
75	0	5.0	3480	1440	5.9
"	10	4.8	3080	1100	0.9
"	20	4.2	3080	920	-2.4
"	30	3.5	2920	460	-5.8
"	40	2.5	2260	170	-7.2
60	0	4.5	2080	800	11
"	10	4.3	1788	630	2.5
"	20	3.2	1350	280	-1.3
"	30	2.9	1038	150	-7.0
"	40	2.3	950	100	-9.6
45	0	4.0	1100	540	9.9
"	10	3.3	930	360	5.1
"	20	2.6	800	250	1.6
"	30	2.3	670	170	-2.5
"	40	2.0	640	40	-7.1
30	0	3.8	780	400	7.8
"	10	3.0	580	260	4.4
"	20	2.6	470	154	1.3
"	30	2.2	424	80	-1.5
"	40	1.9	360	36	-4.1

where s is the shear stress on the lower boundary of the shear zone and p is the mean normal stress acting on that part of a "cranked" idealisation of the lower boundary which is parallel to the cutting speed, V, [8].

If, in fact, our concept of pseudo-orthogonality is valid, then data obtained in single edge free orthogonal cutting *and* data obtained from symmetric vee-cutting of the same work material should both conform to equation (10) (since s, being a material constant [8], must be independent of cutting conditions). In Fig. 4 data from the present work and data published by Armarego [2] are presented and we see that the *same* equation satisfies single edge and vee-cutting data both for our data (s = 170 MPa) and for the data in [2], (s = 240 MPa).

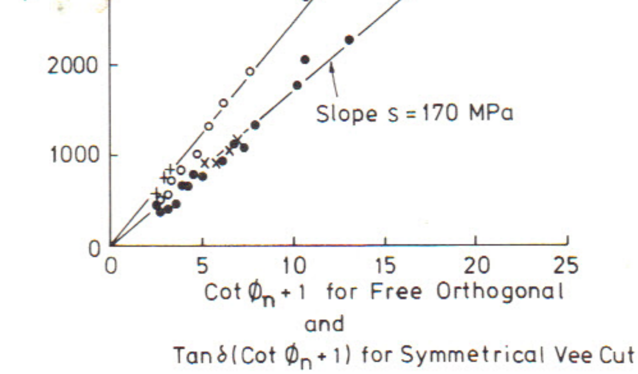

Fig. 4 Verification of Equation (10)

In contrast, our data give values of p (as computed from equation 11) which decrease from +70 MPa (compression) to -400 MPa (tensile) when α_n is changed from 0 to 40^o in single edge orthogonal cutting whereas in vee-cutting, p *remains compressive throughout this range of* α_n, decreasing from +120 MPa to +25 MPa.

These results are quite consistent with our model, for while s is a material constant, p is dependent on cutting conditions - hence, for example, the variation with α_n. Thus if we cut allowing free exit to the chip we can encounter high tensile mean stresses - in contrast, cutting with serious constraint must give rise to compressive stresses which, it turns out, are sufficient in symmetric vee-cutting to convert 400 MPa tensile to 25 MPa compressive. We see then that the origin of F_{in} and the generation of *compressive* values of p *at rake angles as high as* 40^o are to be attributed to the same cause, viz., mutual interference to the flow of the chip segments originating at each cutting edge.

CONCLUSION

This analysis has shown that the concept of twin shear zones in double-edge cutting has been discarded too hastily. A recognition of the ramifications of constrained flow of the chip segments originating at each edge has enabled us to propose a consistent model for symmetrical vee-cutting.

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