REPRINT HMT-115-81

C 14

6th National Heat and Mass Transfer Conference

Organised by

INDIAN INSTITUTE OF TECHNOLOGY

MADRAS

STEADY STATE TEMPERATURES ON OBLIQUELY MOVING HEAT SOURCES

V.V.S. SASTRY
Kakatiya Institute of Technology and Science, Warangal

T.L. SITHARAMA RAO
Regional Engineering College, Warangal

P.K. VENUVINOD
Hong Kong Polytechnic, Hong Kong

ABSTRACT

The subject of moving sources of heat has an extensive application in metal cutting, sliding friction, internal ballastics and numerous metal treatments such as welding, flame hardening etc. In this paper it is attempted to bring out a mathematical model and its numerical solution to determine the temperature at a point in the plane of contact of two sliding solids viz., an obliquely moving source of heat and a semi-infinite solid, under steady state conditions. Equations have been derived to obtain the temperature distribution on a rectangular heat source moving on a semi-infinite solid, the direction of motion being arbitrary. The resulting equation is solved numerically and the results are presented for various aspect ratios of heat source, various velocities and various directions.

NOMENCLATURE

- (x,y) = coordinates of any point at time t
- (x',y') = coordinates of a specific point at specific time t'
- $2\ell, 2m$ = dimensions of a rectangular source
- m/ℓ = aspect ratio of the rectangular source
- v = velocity of the heat source
- α = thermal diffusivity = $K/\rho c$
- K = thermal conductivity
- ρ = density
- c = specific heat
- $L = v\ell/2\alpha$, $M = vm/2\alpha$, $X = vx/2\alpha$, $Y = vy/2\alpha$
- β = angle of obliquity
- $\Delta\theta$ = temperature rise
- Q = quantity of heat liberated by heat source per unit area per unit time
- $G(x,y)$ = non-dimensional temperature rise on the heat source at a point $P(x,y)$
- $\bar{G}$ = non-dimensional average temperature rise on the heat source

INTRODUCTION

The analysis of temperature field in the vicinity of two sliding surfaces in contact, has important applications in many fields such as metal cutting, welding, flame hardening, internal ballastics etc.

In metal cutting process (Fig. 1) the temperature determination became essential to investigate cutting tool life characteristics. Temperature can influence the oxidation rate at the chip-toolinterface, the characteristics of the coolants, the mechanical properties of surface layers of the cut metal, the absorption of gases and lubricants etc.

All the problems such as metal cutting, welding etc., can be mathematically analysed as moving heat source problems. In the mathematical model, the heat source is considered to be plane and to move with constant velocity on plane surface of a semi-infinite solid with no loss of heat from the surface. The temperatures attained in the plane of the sources, when the movement of source has gone on infinitely long i.e., 'quasi-steady state' has been attained, have been discussed, for various types of sources by Carslaw and Jaeger [1] and Jaeger [2].

In metal cutting process the cutting edge of the cutting tool can be simulated to a rectangular heat source and the metal that is being cut, becomes the semi-infinite solid. The aspect ratio has great influence on the temperature level of the moving rectangular heat sources. The aspect ratio when equal to unity reduces the problem to a square source. When $m/l >> 1$, the problem results in a band source (Fig.2) and when $m/l < 1$, it results in a wide source.

THEORETICAL ANALYSIS

Rectangular heat sources have been analysed for orthogonal cutting (Fig.3) by Carslaw and Jaeger [1] theoretically. The steady state temperature rise for a rectangular source ($2\ell \times 2m$) of aspect ratio m/l, moving with a velocity v along a direction parallel to dimension 2ℓ, on the surface of a semi-infinite solid, is obtained as

$$\Delta\theta = \frac{Q\,\alpha}{Kv\,(2\pi)^{1/2}} \int_0^\infty \left[\operatorname{erf}\frac{x + L + W^2}{\sqrt{2}\,W} - \operatorname{erf}\frac{X - L + W^2}{\sqrt{2}\,W}\right]\left[\operatorname{erf}\frac{Y+M}{\sqrt{2}W} - \operatorname{erf}\frac{Y-M}{\sqrt{2}W}\right] dw \qquad (1)$$

Later, Jaeger [2] utilised the above analysis and obtained numerical results for a limited range of values of velocity v. He concentrated on band sources (m/l large) and square sources, in particular and obtained the average and maximum temperatures over the above sources.

Using numerical methods for obtaining solution for (1) for specific aspect ratios and sliding velocities, Reddy [3] has made extensive calculations on the same lines and obtained a universal chart to give the average temperatures over the source area for various values of the aspect ratios (m/l) and sliding velocities of orthogonal movement.

Obliquely moving heat source

Oblique movement of rectangular heat sources has become necessary in recent years as metal cutting operation is not exactly orthogonal. The movement of rectangular heat source is necessarily oblique which means that the velocity vector is inclined to the edge of the rectangular heat source, as shown in Fig. 4.

Analysis of obliquely moving source

Let a rectangular source of dimensions ($2\ell \times 2m$) as in Fig. 4, move on the plane $z = 0$, the plane of contact between the moving source and semi-infinite solid ($z<0$) with a uniform velocity v, along a direction making an angle β with the x-axis (i.e.) with the '2ℓ' - edge of the rectangular source. It is assumed that there is no loss of heat from the plane $Z = 0$, except into the solid ($z<0$) by conduction.

Let a small two dimensional element $dx'\,dy'$ be at $P(x', y')$ at time t' initially.

At time t, the source must have moved a distance of $v\,(t-t')$ from the initial position (x', y') in β-direction, that is, $v_x(t-t')$ along x-direction and $v_y(t-t')$ along y - direction, where $v_x = v \cos\beta$ and $v_y = v \sin\beta$.

Since the co-ordinate system is assumed to be moving along with the source, the new coordinates of the elementary source at time t, would become

$$[x' - v_x\,(t-t'),\; y' - v_y\,(t-t')]$$

Then temperature-rise at any point

(x,y) at time t, due to the above source (dx', dy') is given by [1] as

$$\Delta\Theta = \frac{Q\alpha \, dx' \, dy'}{4 K (\pi\alpha)^{3/2}} \int_{t'}^{t} \frac{dt'}{(t-t')^{3/2}} \exp\left[-\frac{\{x - x' - v_x(t-t')\}^2 + \{y - y' - v_y(t-t')\}^2}{4\alpha (t-t')}\right] \tag{2}$$

Hence temperature-rise at any point (x,y) at time t due to the rectangular source ($2\ell \times 2m$) is given by

$$\Delta\Theta = \frac{Q\alpha}{4 K(\pi\alpha)^{3/2}} \int_{t'}^{t} \frac{dt'}{(t-t')^{3/2}} \int_{-\ell}^{+\ell} dx' \int_{-m}^{+m} dy' \, . \exp\left[-\frac{\{x - x' - v_x (t-t')\}^2 + \{y - y' - v_y (t-t')\}^2}{4\alpha (t-t')}\right] \tag{3}$$

As time $t \to \infty$, a steady state solution is arrived as

$$\Delta\Theta = \frac{Q\,\alpha}{Kv\,(2\pi)^{1/2}} \int_0^\infty \left[\mathrm{erf}\,\frac{X + L + W^2 \cos\beta}{\sqrt{2}\,W} - \mathrm{erf}\,\frac{X - L + W^2 \cos\beta}{\sqrt{2}\,W}\right] . \left[\mathrm{erf}\,\frac{Y + M + W^2 \sin\beta}{\sqrt{2}\,W} - \mathrm{erf}\,\frac{Y - M + W^2 \sin\beta}{\sqrt{2}\,W}\right] dw \tag{4}$$

where $X = \frac{vx}{2\alpha}$, $Y = \frac{vy}{2\alpha}$, $L = \frac{vl}{2\alpha}$, $M = \frac{vm}{2\alpha}$ and $W^2 = \frac{v^2}{2\alpha}(t-t')$

$$\text{or} \quad \Delta\Theta = \frac{Q\,\alpha}{Kv\,(2\pi)^{1/2}} \, . \, G(x,y) \tag{5}$$

Where G(x,y) stands for the integral in (4).

The above equation (4) arrived by the authors reduces to (1) when $\beta = 0$, i.e., the case of orthogonal cutting [1].

SOLUTION

Since explicit mathematical solution is not possible for the integral in (4), numerical methods are adopted. So as to find the temperature-rise at any chosen point on the rectangular source, the integral is to be computed between the limits 0 to ∞. As the expression consists of error functions which are fast converging, Gauss-Laguerre method [4] is adopted. The rectangular source was divided into 441 equal parts by dividing the length and width into 21 parts each. The non-dimensional temperature $G(x,y) = \frac{Kv(2\pi)^{1/2}}{Q\alpha} . \Delta\Theta$ [from (5)] was calculated at the centre of each of these bits using the integral in (4). The average of all these values, $\bar{G}$ was calculated, which denotes the average non-dimensional temperature-rise on the rectangular source. Solutions were obtained fo for the following range of variables.

Aspect ratio (m/l): 0.1 to 20.0

Non-dimensional velocity $L(=vl/2\alpha)$: 0.1 to 100 and

Angle of obliquity (β): 0 to 90

RESULTS AND DISCUSSION

The average temperature $\bar{G}$ on the rectangular source is calculated for the various values of m/l, β and L. The variation of this average temperature as a function of these parameters for some specific values is discussed below.

Influence of aspect ratio (m/l)

Figs.5(a) and 5(b) show the variation of $\bar{G}$ with velocity L for two values of β. The aspect ratios vary from 0.1 to 20 i.e. from a wide source to a band source. The trend of the variation for other values of β is similar.

It is seen that $\bar{G}$ increases with L as well as with m/l for any given value of β. For higher aspect ratios and large L (L>1.0) there is a linear relationship between $\bar{G}$ and L (on log-log scale). For β = 0, the relationship is

$\bar{G} = 3.77\sqrt{L}$, which is the same as for orthogonal band sources [2]. However for different angles of β, the equation is approximated by

$$\bar{G} = 3.77\sqrt{\frac{L}{\cos\beta}}$$

for large m/l (m/l > 20). At lower speeds ($L \leq 1.0$), m/l has a large influence on $\bar{G}$. However at higher speeds the influence of m/l is small and all the curves converge to a straight line (on log-log scale) $\bar{G}=3.77\sqrt{L}$. As β increases, the spread at higher speeds for different values of m/l, becomes large.

Influence of obliquity (Angle β)

The effect of moving a rectangular source at various angles β, is shown in Fig.6. There are 3 sets of graphs for 3 different values of m/l, 0.1, 1.0 and 5.0. It is seen that at lower m/l (wide source) an increase in β reduces the average temperature $\bar{G}$ for a given value of L. However, for a square source (m/l = 1) the angle β has no effect which is obvious.

However, for m/l > 1 (tending to a band source) the effect of increase in β is to increase the average temperature $\bar{G}$ for a given value of L.

But the effect of β is not significant at lower speeds.

Influence of m/l and β

Fig. 7(a) and (b) show the variation of $\bar{G}$ as a function of β with m/l as parameter. At all speeds, $\bar{G}$ is independent of β for a square source. For m/l < 1 (wide source), $\bar{G}$ decreases with increase in β and for m/l>1, a reverse trend is noticed. This can also be concluded by careful observation of Fig. 6. But, it is more clearly shown in Fig. 7.

CONCLUSIONS

(1) All the results and the three families of curves give a wide range of technical data as regards to determination of $\bar{G}$ for any specific values of L, m/l and β.

(2) The plots show the use of attaining very high temperatures by using band sources moving at an angle of 90° or wide sources at an angle 0°, with a special reference to modern metal cutting methods like rotary cutting, hot machining etc. wherein higher rise in temperature is an advantage in easing the cutting process.

(3) The charts give us data for selected variables m/l, L, β for optimum temperature of cutting in ordinary cutting methods.

ACKNOWLEDGEMENTS

The authors wish to thank the Principal, Regional Engineering College, Warangal for providing the facilities and financial assistance for carrying out this work.

REFERENCES

1. Carslaw, H.S. and Jaeger, J.C., "Conduction of Heat in Solids," Oxford University Press, Great Britain.

2. Jaeger, J.C., "Moving Sources of Heat and the Temperature at Sliding Contacts," Journal and Proceedings of the Royal Society, New South Wales, Vol.LXXVI, Part-III, (1942)

3. Reddy, P.N., "Performance of Rotary Tools" Ph.D. Thesis, Kakatiya University,

Warangal, (1977)

4. Abramowitz, M. and Stegun, I.A., "Handbook of Mathematical Functions Dover Publications, Inc., New York.

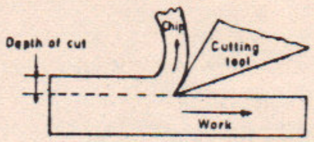

Fig. 1 Metal cutting process

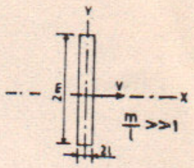

Fig. 2 Band source with orthogonal movement

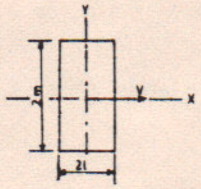

Fig. 3 Rectangular source with orthogonal movement

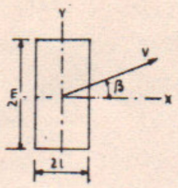

Fig. 4 Rectangular source with oblique movement

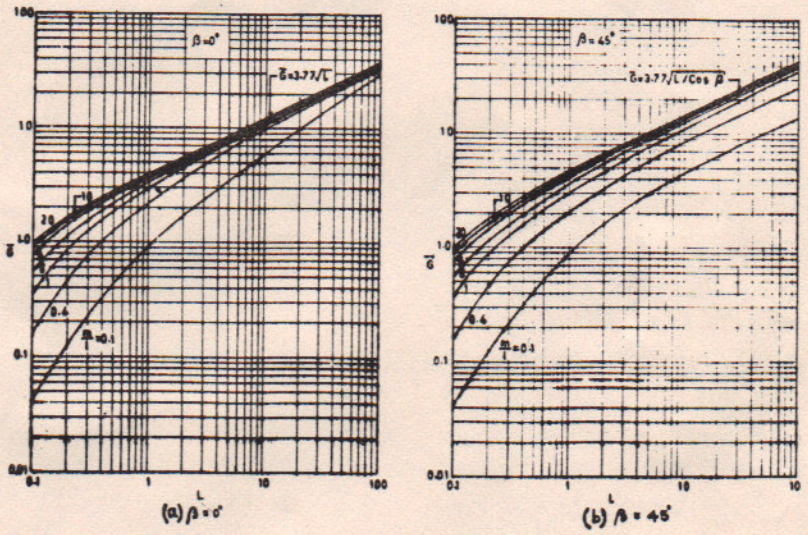

Fig. 5 Influence of aspect ratio

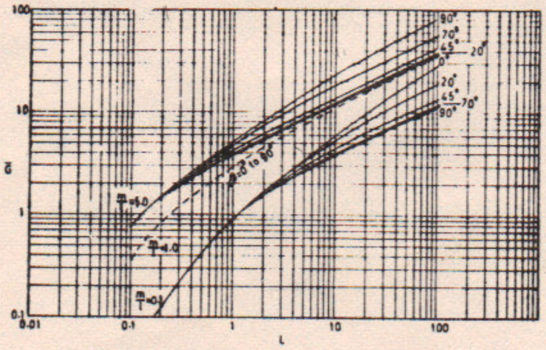

Fig. 6 Influence of obliquity, β

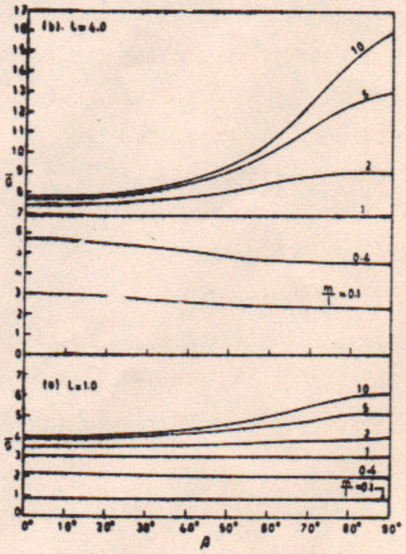

Fig. 7 Influence of m/l and β